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ERRATUM TO "OPERADS IN ITERATED MONOIDAL CATEGORIES," J. OF HOMOTOPY AND RELATED STRUCTURES, VOL.2 (1), 2007, PP.1 - 43

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(*communicated by James Stasheff*)

Abstract

Theorem 5.4 of the paper [1] in question is incorrect as stated. In fact we state here instead that the characterization of minimal 2-fold operads in the natural numbers is an open question.

1. Description of error.

In Example 5.3 of the paper [1] it is pointed out that a nontrivial 2-fold operad in $\mathbb{N}$ is a nonzero sequence $\{\mathcal{C}(j)\}_{j\geqslant 0}$ of natural numbers which has the property that for any $j_1 \dots j_k$, $\max(\mathcal{C}(k), \sum \mathcal{C}(j_i)) \leqslant \mathcal{C}(\sum j_i)$ and for which $\mathcal{C}(1) = 0$.

Recall that minimal examples are formed by choosing a starting term or terms and then determining each later n^{th} term. These are minimal in the sense that the principle which determines each of the later terms in succession is that of choosing the minimal next term out of all possible such terms. For a starting finite sequence $0, a_2, \dots, a_l$ which obeys the axioms of a 2-fold operad so far, the operad $\mathcal{C}_{0,a_2,\dots,a_l}$ is found by defining terms $\mathcal{C}_{a_1,\dots,a_l}(n)$ for $n > l$ to be the maximum of all the values of $\max(\mathcal{C}(k), \sum_{i=1}^{k} \mathcal{C}(j_i))$ where the sum of the j_i is n.

In addition to the examples in the text, here is another.

$$C = \mathcal{C}_{0,3,6,8} = (\emptyset, 0, 3, 6, 8, 9, 12, 14, 16, 18, 20 \dots).$$

It is clear from the above example that Theorem 5.4 of [1] is incorrect as stated.

1.1 Theorem (Incorrect)**.** *If "arbitrary" starting terms $0, a_2, \dots, a_k \in \mathbb{N}$ are given (themselves of course obeying the axioms of a 2-fold operad), then the n^{th} term of the 2-fold operad $\mathcal{C}_{0,a_2,\dots,a_k}$ in $\mathbb{N}$ obeys*

$$a_n = a_q + pa_k \text{ where } n = pk + q, \text{ for } p \in \mathbb{N}, 0 \leqslant q < k.$$

The incorrect theorem would predict that $C(5) = 8, C(6) = 11, C(7) = 14, C(8) = 16, C(9) = 16$ and $C(10) = 19$.

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The failure of the theorem's proof is in faulty induction in the step purporting to prove the inequality $a_n \leqslant a_q + pa_k$. However the other inequality, originally proven first, still holds:

1.2 Theorem. *If "arbitrary" starting terms $0, a_2, \ldots, a_k \in \mathbb{N}$ are given (themselves of course obeying the axioms of a 2-fold operad), then the n^{th} term of the 2-fold operad $\mathcal{C}_{0,a_2,\ldots,a_k}$ in $\mathbb{N}$ obeys*

$$a_n \geqslant a_q + pa_k \text{ where } n = pk + q, \text{ for } p \in \mathbb{N}, 0 \leqslant q < k.$$

Proof. We need to show that

$$a_n = \max_{j_1+\cdots+j_l=n} \{\max(a_l, \sum_{i=1}^{l} a_{j_i})\} = a_q + pa_k$$

where $n = pk + q$, for $p \in \mathbb{N}$, $0 \leqslant q < k$. First we note that $a_q + pa_k$ appears as a term in the overall max, so that $a_n \geqslant a_q + pa_k$. □

Other less strict versions of the theorem may hold as well. For now we leave as an open question the characterization of the minimal 2-fold operads in the natural numbers.

References

[1] S. Forcey, J. Siehler and E. S. Sowers, *Operads in iterated monoidal categories*, Journal of Homotopy and Related Structures, vol.2 (1), 2007, pp.1 - 43

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