Applications of welded tangleoids to effective quantum field theories (EQFT)

Cenap Ozel[1], Hadeel Albeladi[2] and Patrick Linker[3]

[1]Department of Mathematics, King Abdulaziz University, Jeddah 21589 Saudi Arabia
[2] Department of Mathematics, College Science & arts King Abdulaziz University, Rabigh 21911, Saudi Arabia
[3]University Fachbereich Kontinuumsmechanik, TU Darmstadt, 64287 Darmstadt, Germany

E-mail: cenap.ozel@gmail.com[1], aealbeladi@kau.edu.sa[2] ,mrpatricklinker@gmail.com[3]

Abstract

In this paper we show that the welded tangloids show us even macroscopic objects and their dynamics can be characterized by EQFTs. It may also require inclusion of scattering cluster quasiparticles for accurate description, for example in far-from equilibrium processes, because time-dependent dynamics strongly matters.

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1 Introduction

We show an application of welded tangloids to effective quantum field theories. Effective quantum field theories (EQFTs) are field theories for composite particles derived from more fundamental quantum field theories (QFTs). In the literature, effective field theories were derived by integrating in a path integral about the degree of freedom that do not matter in theoretical description [4], [5]. For example, one can derive an EQFT for protons or neutrons that are composed on three quarks from Quantum chromodynamics, the fundamental QFT for quarks and gluons, the elementary particles that make up protons or neutrons.
The paper's organization is as follows:
In section 2, we review the definition of welded tangle-oids categories defined in [1]. A monoidal category (see for example [2]) of unoriented welded tangle-oids have defined by giving a presentation by using presentation of slideable $\frac{1}{2}$-monoidal categories [1].

In Section 3 we show how to the welded tangloids show us even macro- scopic objects and their dynamics can be characterized by EQFTs. It may also require inclusion of scattering cluster quasiparticles for accurate descrip- tion, for example in far-from equilibrium processes, because time-dependent dynamics strongly matters.
Finally we conclude by our main result in last section.

2 Unoriented welded tangle-oids

In this section we review the definition of welded tangle-oids categories defined in [1]. A monoidal category (see for example [2]) of unoriented welded tangle-oids have defined by giving a presentation by using presentation of slideable $\frac{1}{2}$-monoidal categories [1].

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Definition 2.1. [1, definition 7.2.1] Consider the monoidal graph

$$\beta = (\mathbb{N}, E(\beta), \otimes_0, 0, \delta_1, \delta_2),$$

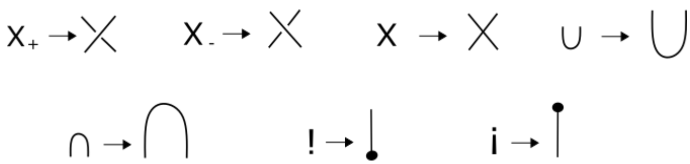
where for all $m, n \in \mathbb{N}$, $m \otimes_0 n = m + n$, and

$$E(\beta) = \{X_+, X_-, X, \cup, \cap, \text{¡}, !\},$$

the incidence maps

$$\begin{array}{rrrr} \delta_1 X_+ = 2, & \delta_2 X_+ = 2, & \delta_1 X_- = 2, & \delta_2 X_- = 2, \\ \delta_1 X = 2, & \delta_2 X = 2, & \delta_1 \cup = 0, & \delta_2 \cup = 2, \\ \delta_1 \cap = 2, & \delta_2 \cap = 0, & \delta_1 \text{¡} = 1, & \delta_2 \text{¡} = 0, \\ \delta_1 ! = 0, & \delta_2 ! = 1. & & \end{array}$$

These generators can be presented geometrically as



Consider the path category, see for example ([3], over β^*, the extent of the monoidal graph β.

$$P(\beta^*) = (\mathbb{N}, \hom_{P(\beta^*)}(n, m), \bullet, \varphi_{\text{-}}).$$

Therefore

$$\Omega(\beta) = (P(\beta^*), \otimes_0, 0, {}_n\#, \#_m)$$

is a $\frac{1}{2}$-monoidal category, whose set of objects is the set of natural numbers, where for all $n, m, k \in \mathbb{N}$;

$${}_n\#_m(k) = n \otimes_0 k \otimes_0 m = n + k + m,$$

and for all generating morphism $(f : k \to k') \in E(\beta)$, we have

$${}_n\#_m(f) = n + k + m \xrightarrow{n \Theta f \Theta m} n + k' + m.$$

Then we have the free-$\frac{1}{2}$-monoidal category-triple

$$(\beta, \Omega(\beta), \delta).$$

Definition 2.2 (Unoriented welded tangle-oids category)**.** The *unoriented welded tangle-oids category $UWTC$* is the strict monoidal category formally presented by

$$\mathfrak{F}\Big(\Omega(\beta)\Big/\overline{\underline{W}}\Big),$$

where $\Omega(\beta)$ defined in [1, Section 7.2] and $\overline{\underline{W}}$ is the $\frac{1}{2}$-monoidal closure of the congruence template W that is defined as follows.

Given $m, n \in \mathbb{N}$, then $W_{m,n}$ is the relation in $\hom_{P(\beta^*)}(m,n)$, defined as (the picture will follow)

In $\hom_{P(\beta^*)}(1,1)$, we have the only relations

- $[WT_1]:\ (\mathrm{id}_1 \otimes \cap)(X \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup) \sim_{W_{1,1}} \mathrm{id}_1 \sim_{W_{1,1}} (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X)(\cup \otimes \mathrm{id}_1)$.
- $[WT_2]: (\mathrm{id}_1 \otimes \cap)(X_+ \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup) \sim_{W_{1,1}} \mathrm{id}_1 \sim_{W_{1,1}} (\mathrm{id}_1 \otimes \cap)(X_- \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup)$.
- $[WT_3]: (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_-)(\cup \otimes \mathrm{id}_1) \sim_{W_{1,1}} \mathrm{id}_1 \sim_{W_{1,1}} (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_+)(\cup \otimes \mathrm{id}_1)$.
- $[WT_4]: (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup) \sim_{W_{1,1}} \mathrm{id}_1 \sim_{W_{1,1}} (\mathrm{id}_1 \otimes \cap)(\cup \otimes \mathrm{id}_1)$.

In $\hom_{P(\beta^*)}(2,2)$, we have the only relation

- $[WT_5]: X_-X_+ \sim_{W_{2,2}} \mathrm{id}_2 \sim_{W_{2,2}} X_+X_-$.

In $\hom_{P(\beta^*)}(3,3)$, we have the only relations

- $[WT_6]: (X_+ \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_+)(X_+ \otimes \mathrm{id}_1) \sim_{W_{3,3}} (\mathrm{id}_1 \otimes X_+)(X_+ \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_+)$.
- $[WT_7]: (X_+ \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X)(X \otimes \mathrm{id}_1) \sim_{W_{3,3}} (\mathrm{id}_1 \otimes X)(X \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_+)$.
- $[WT_8]: (X \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_+)(X_+ \otimes \mathrm{id}_1) \sim_{W_{3,3}} (\mathrm{id}_1 \otimes X_+)(X_+ \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X)$.

In $\hom_{P(\beta^*)}(3,1)$, we have the only relations

- $[WT_9]: (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_-) \sim_{W_{3,1}} (\mathrm{id}_1 \otimes \cap)(X_+ \otimes \mathrm{id}_1)$.
- $[WT_9]': (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X_+) \sim_{W_{3,1}} (\mathrm{id}_1 \otimes \cap)(X_- \otimes \mathrm{id}_1)$.
- $[WT_9]'': (\cap \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes X) \sim_{W_{3,1}} (\mathrm{id}_1 \otimes \cap)(X \otimes \mathrm{id}_1)$.

In $\hom_{P(\beta^*)}(1,3)$, we have the only relations

- $[WT_{10}]: (\mathrm{id}_1 \otimes X_+)(\cup \otimes \mathrm{id}_1) \sim_{W_{1,3}} (X_- \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup)$.
- $[WT_{10}]': (\mathrm{id}_1 \otimes X_-)(\cup \otimes \mathrm{id}_1) \sim_{W_{1,3}} (X_+ \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup)$.
- $[WT_{10}]'': (\mathrm{id}_1 \otimes X)(\cup \otimes \mathrm{id}_1) \sim_{W_{1,3}} (X \otimes \mathrm{id}_1)(\mathrm{id}_1 \otimes \cup)$.

In $\hom_{P(\beta^*)}(1,0)$, we have the only relation

- $[WT_{11}] : \cap(\mathrm{id}_1 \otimes !) \sim_{W_{1,0}}$ ¡ $\sim_{W_{1,0}} \cap(! \otimes \mathrm{id}_1)$.

In $\hom_{P(\beta^*)}(0,1)$, we have the only relation:

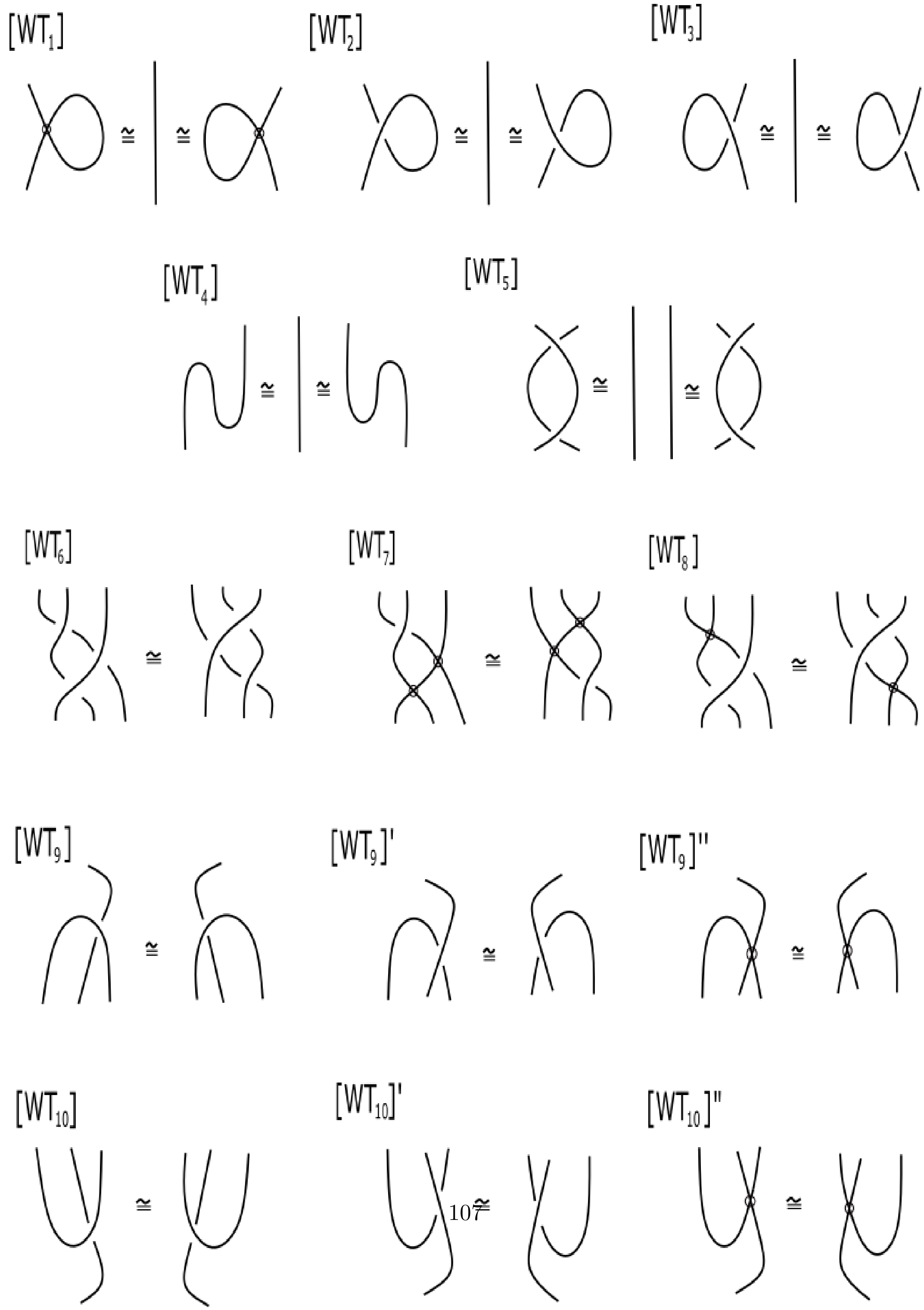
- $[WT_{12}] : (\mathrm{id}_1 \otimes$ ¡$)\cup \sim_{W_{0,1}} \, ! \sim_{W_{0,1}}$ (¡ $\otimes \, \mathrm{id}_1)\cup$.

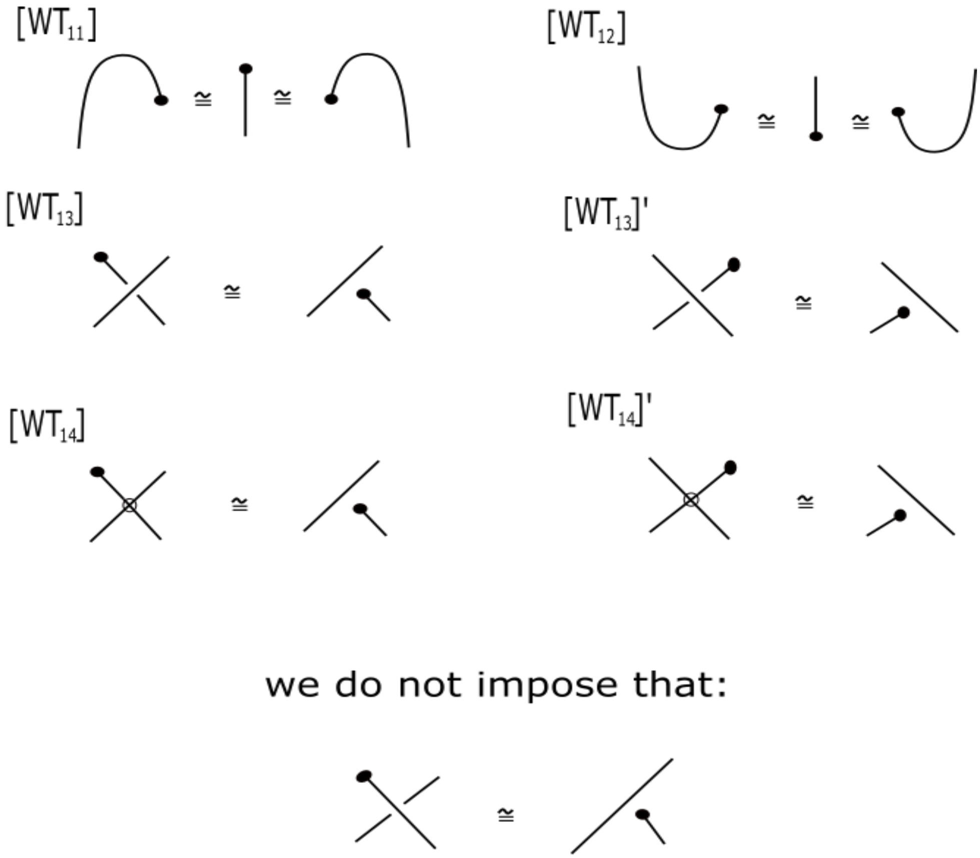
In $\hom_{P(\beta^*)}(2,1)$, we have the only relations

- $[WT_{13}] :$ (¡ $\otimes \, \mathrm{id}_1)X_+ \sim_{W_{2,1}} \mathrm{id}_1 \otimes$ ¡.
- $[WT_{13}]' : (\mathrm{id}_1 \otimes$ ¡$)X_- \sim_{W_{2,1}}$ ¡ $\otimes \, \mathrm{id}_1$.
- $[WT_{14}] :$ (¡ $\otimes \, \mathrm{id}_1)X \sim_{W_{2,1}} \mathrm{id}_1 \otimes$ ¡.
- $[WT_{14}]' : (\mathrm{id}_1 \otimes$ ¡$)X \sim_{W_{2,1}}$ ¡ $\otimes \, \mathrm{id}_1$.

Note that we do not impose that in $\hom_{P(\beta^*)}(2,1)$**:**
(¡ $\otimes \, \mathrm{id}_1)X_- \nsim_{W_{2,1}} \mathrm{id}_1 \otimes$ ¡.

These relations can be present geometrically as (note we read the diagram from bottom to top)





3 Applications of welded tangloids

We show an application of welded tangloids to effective quantum field theories. Effective quantum field theories (EQFTs) are field theories for composite particles derived from more fundamental quantum field theories (QFTs). For example, one can derive an EQFT for protons or neutrons that are composed on three quarks from Quantum chromodynamics, the fundamental QFT for quarks and gluons, the elementary particles that make up protons or neutrons. Elementary particles can be described by a set of N quantum fields $\theta_k, k \in \{1, \dots, N\}$. If the action functional is denoted by S , the path integral for the dynamics of these elementary particles has the form:

$$Z = \int (\prod_{k=1}^{N} D[\theta_k]) e^{iS(\theta_1, \dots, \theta_n)} \quad \dots \qquad (a)$$

Now we can introduce M new effective quantum fields $x_k, k \in \{1,\ldots,M\}$ that can be represented as a function of the elementary quantum fields θ_k, i.e. in the form $x_k = f_k(\theta_1,\ldots,\theta_N)\, k \in \{1,\ldots,M\}$. One can express the path integral (Equation (a)) with the constraint that defines effective quantum fields as:

$$Z = \int (\prod_{k=1}^{N} D[\theta_k])e^{iS(\theta_1,\ldots,\theta_N)}(\prod_{k=1}^{M} D[y_k]e^{iy_k(x_k - f_k(\theta_1,\ldots,\theta_N))}) =$$

$$\int \prod_{k=1}^{M} D[y_k]\left[(\prod_{k=1}^{N} D[\theta_k)])e^{(iS(\theta_1,\ldots,\theta_n)+\sum_{j=1}^{M} iy_j(x_j - f_j(\theta_1,\ldots,\theta_N))}\right] =$$

$$\int \prod_{k=1}^{M} D[y_k]e^{iS_{eff}(y_1,\ldots,y_M)}. \quad ... \quad (b)$$

In Equation (b) we have introduced the effective action S_{eff} that is defined as

$$S_{eff} = -i\ln(\int \left[(\prod_{k=1}^{N} D[\theta_k)])e^{(iS(\theta_1,\ldots,\theta_n)+\sum_{j=1}^{M} iy_j(x_j - f_j(\theta_1,\ldots,\theta_N))}\right]) \quad ....(c)$$

where the functions $S(\theta_1,\ldots,\theta_N)$ and $f_j(\theta_1,\ldots,\theta_N)$ are polynomial and the action $S(\theta_1,\ldots,\theta_N)$ has the form

$$S(\theta_1,\ldots,\theta_N) = \sum_{j,k=1}^{N} c_{jk}\theta_j\theta_k + S_{int}(\theta_1,\ldots,\theta_N) \quad \ldots (d)$$

where, c_{jk} are constants and S_int is the interaction part of the action that has cubic or higher powers in fields θ_i. We will do a Taylor expansion in Equation (c) around variables.

$$S_{eff} = -i\ln(\int \left[(\prod_{k=1}^{N} D[\theta_k)])e^{i\sum_{j,k=1}^{N} c_{jk}\theta_j\theta_k + \sum_{j=1}^{M} iy_jx_j}\right] \sum_{m=0}^{\infty} \frac{{S_{int}}^m(\theta_1,\ldots,\theta_N)}{m!}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}(\sum_{j=1}^{M} -iy_j(f_j(\theta_1,\ldots,\theta_N))^n) \quad ... \quad (e)$$

Upon evaluation of the path integral (Equation (e)), one has to do a contraction of pairs in θ_k fields. To resemble the categorial structure discussed in previous sections, we will analyze first, what contraction of terms with $m = 0$ (i.e. without interaction terms) will give. A contraction will be of generator type $\cup, \cap$, that leading just from one field to another within a polynomial depicted by $f_j(\theta_1,\ldots,\theta_N)$ (same indices j) or within different $f_j(\theta_1,\ldots,\theta_N)$-polynomials (different indices j). Moreover, the X_+, X_- may also depict these contractions. Distinction between $\cup, \cap$ and X_+, X_- will be time ordering: X_+, X_- will occur if fields on different times will be contracted, $\cup, \cap$ are contraction on equal times. In case of non-contracted variables, we have generators $!, i$, open ends in graphs. This happens, if elementary quantum fields still matter in effective theories like when treating atoms or molecules, but electron and/or photon (are particles arising in the more fundamental theory of Quantum Electrodynamics) dynamics still matter. Finally, we have the general case with $m \neq 0$, where interaction vertices matter. Here, the generator X will come into play. It is a contraction of a quartic vertex. Cubic vertices are also in X, when two of its four open ends are identified to be equal. It is a contraction of a quartic vertex. Cubic vertices are also in , when two of its four open ends are identified to be equal. A contraction will be of generator type , that leading just from one field to another within a polynomial depicted by (same indices) or within different -polynomials (different indices). Moreover, the may also depict these contractions. Distinction between and will be time ordering: will occur if fields on different times will be contracted, are contraction on equal times. In case of non-contracted variables, we have generators , open ends in graphs. This happens, if elementary quantum fields still matter in effective theories like when treating atoms or molecules, but electron and/or photon (are particles arising in the more fundamental theory of Quantum Electrodynamics) dynamics still matter. Finally, we have the general case with , where interaction vertices matter. Here, the generator will come into play. It is a contraction of a quartic vertex. Cubic vertices are also in , when two of its four open ends are identified to be equal. So this category will resemble So this category will resemble all possible contractions performed when transitioning from a fundamental theory with up to quartic vertices to an effective theory. It distinguishes between equal-time contractions (pure compositions of elementary particles, e.g. atoms and molecules) and different-time contractions (quasiparticles which carry information on specific dynamic behavior, e.g. clusters linked to certain scattering processes).

4 Conclusion

In this paper the welded tangloids show us that even macroscopic objects and their dynamics can be characterized by EQFTs. It may also require inclusion of scattering cluster quasiparticles for accurate description, for example in far-from equilibrium processes, because time-dependent dynamics strongly matters.

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